國立成功大學八十三學年度轉學生考試(微積分試題)共 1 頁 第 1 頁

1. (a) Let $[x]$ denote the greatest integer function on $\mathbb{R}$. Show that $\lim_{x\to\infty}\frac{[x]^2}{x}=\infty$. 8%

   (b) Is $\sum_{n=2}^{\infty}\frac{1}{n\ln n}$ convergent ? 7%

2. (a) Suppose $a>0$. Evaluate $\int_0^a \frac{e^x}{e^x+e^{a-x}}\,dx$. 7%

   (b) Suppose $a, b>0$ and $R$ is the region $\left\{(x,y): \frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1\right\}$ in the plane. Evaluate $\iint_R \left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)dxdy$. 8%

3. Find the limit as $x$ approaches 0 of the ratio of the area of the triangle to the total shaded area in Figure 1. 10%

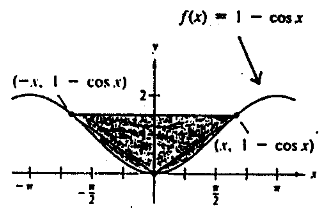


Figure 1

4. (a) Find the points on the paraboloid $z=4x^2+y^2$ at which the tangent plane is parallel to the plane $x+2y+z=6$. 9%

   (b) Suppose that a particle moving on a metal plate in the $xy$-plane has velocity $\vec{v}=\langle 1,-4\rangle$ (cm/sec) at the point $(3,2)$. If the temperature of the plate at points in the $xy$-plane is $T(x,y)=y^2\ln x$, $(x\ge 1)$, in degrees Celsius, find $\frac{dT}{dt}$ at $(3,2)$, where $t$ denotes time. 8%

5. Suppose $f:[0,\infty)\to\mathbb{R}: f(x)=\int_2^x\frac{dt}{\sqrt{1+t^4}}$.

   (a) Show that $f$ is a one-to-one function. 5%

   (b) Let $f^{-1}$ denote the inverse of $f$. Find $(f^{-1})'(0)$. 10%

6. Let $\Gamma$ be the circle with radius 3 and center at the origin. A particle travels once around $\Gamma$ in counterclockwise direction under the force field $\vec{F}(x,y)=\langle y^3, x^3+3xy^2\rangle$. Use Green's theorem to find the work done by $\vec{F}$. 15%

7. In the plane let $L$ be a line and $\Gamma$ an ellipse that forms the boundary of a bounded region $K$. Use the intermediate-value theorem to show that there is a line parallel to $L$ that cuts $K$ into two pieces of equal area. 13%