國立成功大學應用數學所 數値分析 博士班資格考
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1. Let x be a floating point number in IEEE double precision arithmetic satisfying $1 \le x < 2$. Show that $f\ell(x * (1/x))$ is either 1 or $1 - \epsilon/2$, where $\epsilon = 2^{-52}$ (the machine epsilon). (15%)

2. Let x, y be two non-zero vectors in $\mathbb{R}^n$. Show that there is always a Householder transformation $H \in \mathbb{R}^{n\times n}$ such that $Hx = y$. (15%)

3. Suppose the perturbation $\delta(t)$ is proportional to t, that is, $\delta(t) = \delta t$ for some constant δ. Show directly that the following initial-value problem

$$\begin{aligned} &y' = 1 - y,\ 0 \le t \le 2,\\ &y(0) = 0 \end{aligned}$$

   is well-posed. (15%)

4. Let $A(\epsilon) \in \mathbb{R}^{n\times n}$ be a real matrix function of the real variable ϵ, that is, each element of matrix $A(\epsilon)$ is a function of ϵ. Show that the eigenvector $x(\epsilon)$ corresponding to a multiple eigenvalue $\lambda(\epsilon)$ can be not differentiable with respect to ϵ whenever all elements of matrix $A(\epsilon)$ and the eigenvalue $\lambda(\epsilon)$ are differentiable. (15%)

5. Calculate $\sqrt[5]{33}$ upto 4 digits after the decimal point and estimate the error bound of your answer. (15%)

6. Consider the initial-value problem

$$\begin{aligned} &y' = f(t, y),\ a \le t \le b,\\ &y(a) = \alpha \end{aligned}$$

   Let $h = (b - a)/N$. Show that the difference method

$$\begin{aligned} &w_0 = \alpha,\\ &w_{i+1} = w_i + a_1 f(t_i, w_i) + a_2 f(t_i + \alpha_2, w_i + \delta_2 f(t_i, w_i)), \end{aligned}$$

   for each $i = 0, 1, \ldots, N-1$, can not have local truncation error $O(h^3)$ for any choice of constants a_1, a_2, α_2 and δ_2. (15%)

7. Let $A = \begin{bmatrix} 0 & 1 \\ a & b \end{bmatrix}$ be a 2×2 real matrix. Suppose that A has only a double eigenvalue. Show that A must be similar to a Jordan matrix. (10%)