Numerical Analysis 2006/2/23

1. Use Householder's method to place the following matrix in tridiagonal form:

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & -1 \end{bmatrix}$$

2. The set of Legendre plynomials, P_n, is orthogonal on $[-1,1]$ with respect to the weight function $w(x) = 1$. Use the Gram–Schmidt process to obtain that $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = x^2 - \frac{1}{3}x$, and $P_3(x) = x^3 - \frac{3}{5}x$.

3. If A is a strictly diagonally dominant matrix, then for any choice of $\mathbf{x}^{(0)}$, the Jacobi method gives sequences $\{\mathbf{x}^{(k)}\}_{k=0}^{\infty}$ that converge to the unique solution of $A\mathbf{x} = \mathbf{b}$.

4. Write a computer program in C or FORTRAN for the backward substitution algorithm solving the upper triangular system $U\mathbf{x} = \mathbf{b}$, where U is an upper triangular matrix.

5. Factor the following matrix into the LU decomposition with $l_{ii} = 1$ for all i:

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix}$$

6. Show that a second–order Runge–Kutta formula of the form

$$x(t+h) = x(t) + w_1 h f(t,x) + w_2 h f(t + \alpha h, x + \beta h f)$$

must impose these conditions:

$$\begin{aligned} w_1 + w_2 &= 1 \\ w_2 \alpha &= \tfrac{1}{2} \\ w_2 \beta &= \tfrac{1}{2} \end{aligned}$$

7. Find the constants A_0, A_1, A_2, x_0, x_1, and x_2 such that the Gaussian quadrature rule

$$\int_{-1}^{1} f(x)\,dx \approx A_0 f(x_0) + A_1 f(x_1) + A_2 f(x_2)$$

is exact for f in Π_5, which the set of polynomials of degree less than or equal to 5.

8. Use the extended Newton divided difference method to obtain a polynomial p that takes these values:

$$p(0) = 2 \quad p'(0) = -9 \quad p(1) = -4 \quad p'(1) = 4 \quad p(2) = 44.$$