分析通論(博士班)

91.9.20
AM 9:00-12:00

1. (15%) If $f(x)$ is absolutely continuous in $[a, b]$, prove that $|f(x)|^p, p > 0$ *is* absolutely continuous in $[a, b]$.

2. (15%) Let f be a real-valued, measurable function on $\mathbf{R}$ that satisfies the equation

$$f(x+y)=f(x)+f(y)$$

for all x,y in $\mathbf{R}$. Prove that $f(x) = Ax$ for some number A..

3. (20%) Prove or disprove
   (a) The Dirichlet function is measurable.
   (b) If $f(x)$ is Riemann integrable in $[a, b]$, then $f(x)$ must be measurable.
   (c) There exist non-measurable functions.

4. (15%) Prove or disprove. If $f_n \to f$ a.e. then $f_n \to f$ in L^2

5. (20%) Show that if $f,g \in C(\mathbf{R};\mathbf{C})$and for all x;, $f(x+1) = f(x), g(x+1) = g(x)$, then

$$\lim_{n\to\infty}\int_0^1 f(x)g(nx)dx = (\int_0^1 f(x)dx)(\int_0^1 g(x)dx)$$

Use this result to show

$$\lim_{n\to\infty}\int_0^{2\pi} f(x)\sin nx dx = 0 \quad , \forall f \in C([0,2\pi];\Re)$$

6. (15%) If $f \in L(\mathbf{R^n})$ and g is bounded and uniformly continuous on $\mathbf{R^n}$, then the convolution $f * g$ is bounded and uniformly continuous on $\mathbf{R^n}$.