機率論

(碩士班)

91.9.20
PM 1:00 - 4:00

1. Let $(An)n\geq 1$ be an independent sequence of sets with $\sum_{n=1}^{\infty} P(An) = +\infty$. Find

$$\lim_{n\to+\infty} \frac{\sum_{j=1}^{n} 1A_j(w)}{\sum_{j=1}^{n} P(A_j)} \quad \text{where } 1_A(w) = \begin{cases} 1, & w \in A \\ 0, & w \notin A \end{cases}$$

and prove it. (25%)

2. Let Xn have the binomial distribution with parameter (n,p_n), and suppose that $np_n \to \lambda \geq 0$. Prove that Xn converges in distribution to the Poisson d.f. with parameter λ. (25%)

3. Let $(X_n)_{n\geq 1}\ X$ be random variables and suppose that $X_n \to X$ in probability.

L1

Show that $f\{X_n\} \xrightarrow{L^1} f(X)$ for all bounded and uniformly continuous function $f: \boldsymbol{R} \to \boldsymbol{R}$. (25%)

4. Let $(X_n)_{n\geq 1}$ be independent, identically distributed with mean 0 and variance σ^2, $0 < \sigma^2 < +\infty$, Let $S_n = X_1 + X_2 + \ldots + X_n$.

Find

$$\lim_{n\to+\infty} E\left(\frac{S_n^-}{\sqrt{n}}\right)$$

and prove it. (25%)