Oper. Res. Qualifying Exam Fall 2002

(1) Consider the following Linear Programming problem:

(**P**) min $3x_1 + x_2 + 2x_3$

s.t. $2x_1 + x_2 + 3x_3 = 5$

$4x_1 + x_2 + x_3 = 4$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

(i) Formulate the dual of (**P**). 5%

(ii) Formulate the Phase(I) problem. 5%

(iii) Solve the Phase(I) problem. 10%

(iv) Identify the initial basic variables of (**P**) from the Phase(I) result. 5%

(v) Write down the simplex multiplier corresponding to the above initial basic solution. 5%

(vi) Draw a picture of the feasible domain of (**P**) by projecting onto the subspace containing all initial nonbasic variables. 5%

(vii) Identify all good directions along which the objective value of (**P**) can be decreased. 5%

(viii) When there is any good direction to further decrease the objective value, is the simplex multiplier (obtained in (v)) dual feasible? Why? 5%

(ix) Interpret the connection between good directions and reduced cost vectors. 5%

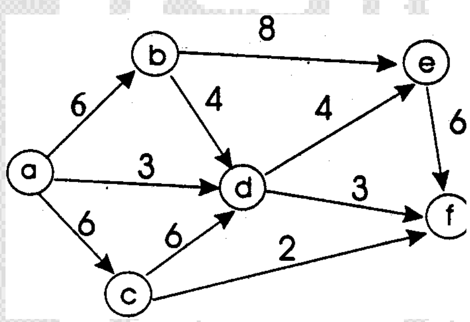
(x) Write down the optimal solution set of (**P**). 5%

(xi) State the Fundamental Theorem of Linear Programming. 5%

(xii) Explain how the idea of the simplex method is based on the Fundamental Theorem of Linear Programming. 10%

(2) State and prove the weak duality theorem of Linear Programming. 10%

(3) Given the following network:



(i) Find the dual solution that corresponds to the cut $K = (S, \bar{S})$ with $S = \{a,b,d\}$ aad $\bar{S} = \{c,e,f\}$. 5%

(ii) State the max-flow-min-cut theorem. 5%

(iii) Find the minimum cut of the network. 5%

(iv) Use the first-labelling-first-picked strategy and start from the arc (A, C) to find the maximal flow of this network. That is, if $a_1 \in (S, \bar{S})$ and $a_2 \in$ S $(S, \bar{S})$ are both unsaturated edges and create no cycle, but the tail of a_1 was labelled before that of a_2, we pick a_1 to continue the search. 10%