91年度碩士班資格考，科目:數值分析　題目共七題有二頁，總分:100

2002年9月20日

1. Let $f \in \underline{C}^3(a,b)$ and $|f'''(x)| < M$ for $x \in (a,b)$. Consider a centered difference formula to be an approximation of $f'(x)$, i.e.,

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}, x \in (a.b)$$

Show that the centered difference formula is numerical unstable, i.e., the error function $e_{x},(h)$ which is defined by the difference of the derivative $f'(x)$ and the approximation formula satisfies

$$e_x(h) \leq \frac{\varepsilon}{h} + \frac{h^2}{6}M,$$

where $\varepsilon = \max\{|f(x+h) - f\lambda(f(x+h))|, |f(x-h) - f\lambda(f(x-h))|\}$.
10%

2. Show that there is a unique quadratic function Pt satisfying the conditions

$$p_2(0) = a_0, \quad p_2(l) = a_1 \quad \text{and} \quad \int_0^1 p_2(x)\,dx = \bar{a}$$

with given a_0, a_1 and $\bar{a}$. 10%

3. Consider the nonlinear integral equation

$$u(t) = \int_0^1 k(t,s,u(s))ds$$

over the space $U == C[0,1]$. Assume $k \in C([0,1] \times [0,1] \times IR)$ and is continuously differentiable with respect to its third argument. Introducing an operator $F : U \to U$ through the formula

$$F(u)(t) = u(t) - \int_0^1 k(t,s,u(s))ds, \quad t \in [0,1],$$

the integral equation can be written in form $F(u) == 0$.

(i) Describe a Newton-type method to solve the nonlinear integral equation, 10%

(ii) Explore sufficient conditions for the convergence of the Newton-type method, 10%

4. Is it possible to use $af(x+h) + bf(x) + cf(x-h)$ with suitably chosen coefficients a, b, c to approximate $f'''(x)$?How many function values are needed to approximate $f'''(x)$?*10%*

5. Show that the QR algorithm applied to a singular unreduced upper Hessenberg matrix must converge in one step. Can you determine the total number of zero eigenvalues at this moment? 10%

6. (i) Show that if $A = M — N$ is singular, then we can never have $\rho\,(M^{-1}\,'N) < 1$ even if M is nonsingular, 10%

(ii) Compare $\rho\,(M_J^{-1} N_J)$ and $\rho\,(M_G^{-1} N_G)$ for the matrix

$$A=\begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix}$$

Here $A = M_J — N_J$ and $A = M_G — N_G$ denote basic splits corresponding to Jacobi and Gauss-Seidel iterations, respectively, 10%

7. (i) Describe the advantage of Conjugate Gradient (CG) method for a symmetric positive definite linear system, 10%

(ii) Explain the role of the preconditioner for the Precondition Conjugate Gradient (PCG) method. Please explain why we need to consider a preconditioner for the CG-iteration. What is the advantage and how to choose a suitable one? 10%