分析通論(碩士班)

91.9.20
AM 9:00-12:00

1. (15%) Prove that tht set [0,1] is not countable by measure theory. Can yon prove this fact by Cantor's diagonal argument?

2. (15%) Let f be a real-valued, measurable function on $\Re$ that satisfies the equation

$$f(x+y) = f(x)+f(y)$$

for all x,y in $\Re$. Prove that $f(x) = A_x$ for some number A. (Hint: Prove this when f is continuous by examining f on the rationals.)

3. (15%) Show that the function $\frac{\sin x}{x}$ is Riemann integrable on $(-\infty,\infty)$ but that its Lebesgue integral does not exists.

4. (10%) If $f \in L(0,1)$, show that $x^k f(x) \in L(0,1)$ for $k = 1,2,\ldots$..and

$$\int_0^1 x^k f(x)dx \to 0$$

5. (15%) Find the limit

$$\lim_{n\to\infty}\int_0^n (1+\frac{x}{n})^n e^{-2x}dx$$

You need to figure out the dominating function.

6. (15%) Let $p > 0$ and $f \in L^p(\mu)$ where $f \geq 0$, and let $f_n = \min(f,n)$. show that $f_n \in L^p(\mu)$ and $\lim_{n\to\infty}\| f - f_n \|_p = 0$

7. (15%) Let $f(x,y)= \frac{xy}{(x^2+y^2)^2}$, $(x,y) \in [-1, 1] \times [-1, 1]$ defining $f(0,0) = 0$

Show that the iterated integrals of f over the square are equal

$$\int_{-1}^{1}\int_{-1}^{1} f(x,y)dxdy = \int_{-1}^{1}\int_{-1}^{1} f(x,y)dydx = ??$$

Is f integrable?