(2017 Spring) **Qualifying Examination**
Subject: Mathematical Statistics

1(E, 20%). Let Z be a standard normal random variable and U be a chi-square random variable with n degree of freedom. Suppose that Z and U are independent. Please find the probability density function of $Z/\sqrt{U/n}$.

2(E, 20%). Let $X_1, ..., X_n$ be independent $N(\mu, \sigma^2)$ random variables. Denote $\overline{X} = \sum_{i=1}^{n} X_i/n$. Prove or disprove that the random variable $\overline{X}$ and the vectors of random variables $(X_1 - \overline{X}, X_2 - \overline{X}, ..., X_n - \overline{X})$ are independent.

3(E, 20%). Let $X_1, ..., X_n$ be a sequence of independent Bernoulli random variables with $P(X_i = 1) = \theta$. Prove or disprove that $T = \sum_{i=1}^{n} X_i$ is a sufficient statistic for θ.

4(M, 20%). Let $x_1, ..., x_n$ be independently sampled from the Poisson distribution with parameter λ. (1) Please derive the maximum likelihood estimate of λ. (2) Find the asymptotic variance of the maximum likelihood estimate. (3) Find the method of moments estimate of λ.

5(D, 20%). Let $X_1, ..., X_n$ be i.i.d. with density function $f(x \mid \theta)$, where f is differentiable. Let $T = t(X_1, ..., X_n)$ be an unbiased estimate of θ. Prove or disprove that $\mathrm{Var}(T) \geq [nI(\theta)]^{-1}$, where $I(\theta) = \mathbb{E}\left[\frac{\partial}{\partial\theta} \ln f(x \mid \theta)\right]^2$.