Question: #	1	2	3	4	5	6	7	total
points:	10	15	15	15	15	15	15	100
Score:								100

Easier: 1, 2, 3, 4 Medium: 5, 6 Harder: 7

(1) Determine whether each statement below is true or false. If true, give a brief explanation. If false, provide a counterexample.
  (a) (5pts) All subgroups of a direct product $G \times H$ of groups G and H are of the form $R \times S$ for $R \leq G$ and $S \leq H$.
  (b) (5pts) An Artinian integral domain is a field.

(2) (15pts) Prove that $\mathbb{Z}[\sqrt{-1}]$ is a principal ideal domain.

(3) (15pts) How many elements of order 7 are there in a simple group of order 168?

(4) (15pts) Show that the set of nilpotent elements (i.e. elements x such that $x^n = 0$, for some non-negative integer n) in a commutative ring R is an ideal.

(5) (15pts) Find the number of monic irreducible quadratic polynomial(i.e. irreducible polynomials of the form $x^2 + ax + b$) in $\mathbb{Z}_p[x]$, where p is a prime.

(6) (a) (10pts) Find all intermediate fields between $\mathbb{Q}$ and $\mathbb{Q}(\sqrt{2}, \sqrt{3})$.
  (b) (5pts) Show that $\sqrt{5}$ is not an element in $\mathbb{Q}(\sqrt{2}, \sqrt{3})$.

(7) (15pts) Show that the ideal generated by 13 and $x^3 - 2$ in $\mathbb{Z}[x]$ is maximal.