## 89 學年度 計算機概論與離散數學

1. Let B, C and D be any natural numbers. Let $a_0 = 1$, $a_1 = 5$, and
   $(\forall n \geq 2)[a_n = 2a_{n-1} + Ba_{n-2} + Cn + D]$. Find natural numbers $B, C, D \geq 10$ such that the following is true: $(\forall n \geq 10)[a_n \equiv 1 \pmod 4)]$. Justify your answer. 10%

2. Seventeen different integers are chosen from 1, 2, 3, ..., 30. Show that at least a pair of the numbers chosen differ by 3. Can you show this for 15 integers? Explain your answer. 10%

3. Consider the following weighted graph.
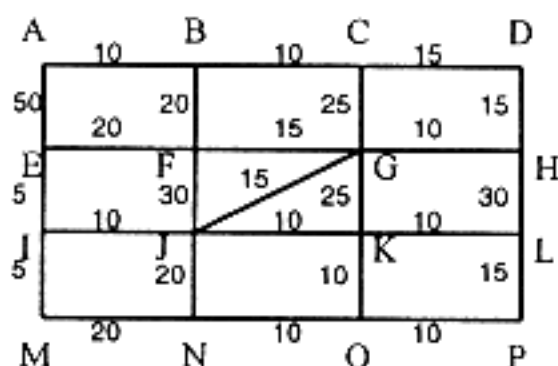
(i) Find a short path from point A to point P. 10%

(ii) Suppose you have a subroutine implementing the Dijkstra's algorithm and a computer equipped with 4 CPUs. Design an algorithm applying the subroutine and the computer to find all the shortest paths from point X to point Y, where $X, Y \in \{A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P\}$. 10%

4. In the following floating point system, a number x has the form $x = \pm 0.b_1 b_2 b_3 b_4 \times 10^e$. Here
   $0.b_1 b_2 b_3 b_4$ is the mantissa. The b_i are base-10 digits and satisfy $0 \leq b_i \leq 9$ for $i = 1, 2, 3, 4$. The exponent satisfies $-99 \leq e \leq 99$.

   (i) Use this floating point system to compute $\sum_{i=1}^{20000} 3$. We assume all integers and real numbers are represented by this floating point system. 10%

   (ii) What is the absolute error of the answer in part (i)? 10%

5. Define $x_{i+1} = f(x_i)$, for $i \geq 0$ and $f(x) = \begin{cases} 2x & \text{if } 0 \leq x < 0.5, \\ 2(1-x) & \text{if } 0.5 \leq x \leq 1. \end{cases}$

   (i) Let $x_0 = 0.2$. Compute the value of x_{2000}. 10%

   (ii) Consider a computer equipped with a Pentium III 733 MHz CPU and implementing IEEE double precision standards to do the arithmetic. What would you expect from the computer for the computed value of x_{2000}? 10%

   (iii) Let the computed value in part (ii) be $\hat{x}_{2000}$. Is $x_{2000} = \hat{x}_{2000}$? If yes, state your reasons; if not, explain how you may improve the accuracy? 10%

6. Suppose you have two computers: computer A has a 64-bit CPU and 128 mega-byte main memory; computer B has a 128-bit CPU and 64 mega-byte main memory. Discuss the advantages and disadvantages of each of the computers in scientific computing. 10%