

### Differential Equations

1. (15%) Consider the equation

$$\frac{d^2x}{dt^2} + \sin x = 0.$$

Sketch the potential and phase planes, and discuss the stability.

2. (15%) Find a formal eigenfunction expansion, if exists, for the solution to the nonhomogeneous boundary value problem

$$y'' + y = \cos 3x + \cos 8\pi; \quad y'(0) = 0, \quad y'(\pi) = 0.$$

3. (14%) Find all the real eigenvalues and eigenfunctions for the eigenvalue problem

$$y'' + 4y + \lambda y = 0; \quad y(0) = 0, \quad y'(\pi) = 0.$$

4. (14%) Compute the Fourier sine series for

$$f(x) = e^x, \quad 0 < x < 1.$$

5. (14%) Solve  $x'(t) = Ax(t)$ , where  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$ .

6. (14%) Use the method of Laplace transform to solve the initial value problem

$$y'' + y = u(t-2); \quad y(0) = 0, \quad y'(0) = 1,$$

where  $u(t)$  is the unit step function.

7. (14%) Solve

$$(y^2 + 2xy) dx + x^2 dy = 0.$$