國立成功大學/1十四學年度為明毅子學考試(機率論試題)等/頁

PROBABILITY

- 1. Suppose (Ω, F, P) is a probability space and X is a random variable (r.v.) defined on Ω into R.
 - (a) State the definition of the probability space (Ω, F, P) . (5%)
 - (b) State the definition of the r.v. X. (5%)
 - (c) Suppose the probability density function (p.d.f.) of the r.v. X is $f_x(x)$. Let $Y=X^2$. Is Y a r.v.? If yes, find the p.d.f. of the r.v. Y, say f_y , in terms of $f_x(\cdot)$. (10%)
- 2. Let Y be a r.v. distributed as an uniform distribution, U(0, 1), and let F be a distribution function (d.f.).

Define the r.v. X by $X=F^{-1}(Y)$, where $F^{-1}(y)=\inf\{x\in R; F(x)\geq y\}$.

- (a) Prove that: $F^{-1}(y) \le t$ if and only if $y \le F(t)$. (10%)
- (b) Prove that: The distribution function of X is F. (10%)
- 3. Suppose that X_1 and X_2 have a continuous joint distribution for which the joint p.d.f. is $f(x_1, x_2)=x_1+x_2$, for $0 < x_1 < 1$, $0 < x_2 < 1$; and zero otherwise. Define $U=X_1X_2$, $V=X_1/X_2$.
 - (a) Prove or disprove that: E(UV)=E(U)E(V). (10%)
 - (b) Prove or disprove that: E(U/V)=E(U)/E(V). (10%)
- 4. Suppose X is a Poisson r.v. with mean 20. Let $p=\Pr\{X\geq 26\}$.
 - (a) Use the Markov inequality to obtain an upper bound on p. (5%)
 - (b) Prove the one-sided Chebyshev inequality. That is, if X is a r.v. with mean 0 and finite variance σ^2 , then for any a>0 $\Pr\{X \ge a\} \le \sigma^2/(\sigma^2 + a^2).(5\%)$
 - (c) Use the one-sided Chebyshev inequality to obtain an upper bound on p. (5%)
 - (d) Use the central limit theorem to approximate p, answer it in terms of $\Phi(\cdot)$ which is the d.f. of the r.v. N(0, 1).(5%)
- 5. Suppose that $X_1, X_2, ..., X_n, ...$ is a sequence of random variables from a distribution with d.f. $F(x)=1-(1+x)^{-1}$, for x>0; and zero otherwise.

Let $Y_n=nX_{(1)}$, where $X_{(1)}=min\{X_1, X_2, ..., X_n\}$.

- (a) Find the d.f. of Y_n . (10%)
- (b) Find the limiting distribution of Y_n, i.e. find the d.f. of a r.v. Y for which Y_n converges in distribution to Y. (10%)