

Calculus Final Exam Solution

Exam Set: C

$$\begin{aligned} 1. (a) \int_0^1 (x+3)\sqrt{2-x} dx &= -\int_0^1 (-x-3)\sqrt{2-x} dx \\ &= -\int_0^1 (2-x-5)\sqrt{2-x} dx \\ &= -\int_0^1 (2-x)^{\frac{3}{2}} - 5(2-x)^{\frac{1}{2}} dx \\ &= \int_0^1 (2-x)^{\frac{3}{2}} - 5(2-x)^{\frac{1}{2}} d(2-x) \\ &= \frac{2}{5}(2-x)^{\frac{5}{2}} - \frac{10}{3}(2-x)^{\frac{3}{2}} \Big|_0^1 \\ &= -\frac{44}{15} + \frac{76\sqrt{2}}{15} \approx 4.2320 \end{aligned}$$

$$\begin{aligned} (b) \int_0^1 \frac{e^{2x}}{e^{2x}+1} dx &= \frac{1}{2} \int_0^1 \frac{1}{e^{2x}+1} d(e^{2x}+1) \\ &= \frac{1}{2} \ln(e^{2x}+1) \Big|_0^1 \\ &= \frac{1}{2} [\ln(e^2+1) - \ln 2] \approx 0.7169 \end{aligned}$$

$$\begin{aligned} (c) \int_0^4 |2x-1| dx &= \int_{\frac{1}{2}}^4 (2x-1) dx + \int_0^{\frac{1}{2}} (1-2x) dx \\ &= x^2 - x \Big|_{\frac{1}{2}}^4 + x - x^2 \Big|_0^{\frac{1}{2}} \\ &= \frac{25}{2} \end{aligned}$$

$$\begin{aligned} (d) \int_0^1 x(x+1)^{10} dx &= \int_0^1 (x+1-1)(x+1)^{10} dx \\ &= \int_0^1 (x+1)^{11} - (x+1)^{10} d(x+1) \\ &= \frac{1}{12}(x+1)^{12} - \frac{1}{11}(x+1)^{11} \Big|_0^1 \\ &\approx 155.159 \end{aligned}$$

$$\begin{aligned} 2. y &= (x^2+1)^{2x+5} \\ \Rightarrow \ln y &= (2x+5) \ln(x^2+1) \\ \Rightarrow \frac{1}{y} \frac{dy}{dx} &= \ln(x^2+1) + (2x+5) \frac{2x}{x^2+1} \\ \Rightarrow \frac{dy}{dx} &= 2[\ln(x^2+1)](x^2+1)^{2x+5} + (4x^2+10x)(x^2+1)^{2x+4} \end{aligned}$$

$$3. \quad (1) \quad \frac{dR}{dx} = 50 - 0.02x + \frac{100}{x+1}$$

$$\Rightarrow R = 50x - 0.01x^2 + 100 \ln |x+1| + C$$

$$x = 0 \quad , \quad R = 0$$

$$\Rightarrow 0 = 100 \ln 1 + C$$

$$\Rightarrow C = 0$$

$$\text{hence } R = 50x - 0.01x^2 + 100 \ln |x+1|$$

$$(2) \quad x = 1500$$

$$R = 50 \times 1500 - 0.01 \times 1500^2 + 100 \times \ln(1500+1)$$

$$= 75000 - 22500 + 731.3887 \approx 53231.3887$$

$$4. \quad \frac{dV}{dt} = 10000(t-6)$$

$$\Rightarrow V = 5000t^2 - 60000t$$

$$V(3) = -135000$$

$$V(0) = 0$$

$$V(3) - V(0) = -135000$$

$$\text{hence loss } 135000$$

$$5. \quad \text{Let } f(x) = \frac{5}{x^3+1}$$

$$\begin{aligned} \int_0^2 \frac{5}{x^3+1} dx &\approx \frac{1}{2} [f(\frac{1}{4}) + f(\frac{3}{4}) + f(\frac{5}{4}) + f(\frac{7}{4})] \\ &\approx 5.4596 \end{aligned}$$

$$6. \quad \int_1^2 (x-1) - (x-1)^3 dx + \int_0^1 (x-1)^3 - (x-1) dx$$

$$= \int_1^2 (x-1-x^3+3x^2-3x+1) dx + \int_0^1 (x^3-3x^2+3x-1-x+1) dx$$

$$= \int_1^2 (-x^3+3x^2-2x) dx + \int_0^1 (x^3-3x^2+2x) dx$$

$$= \frac{1}{2}$$

$$7. \quad (a) \quad \int_0^1 \pi \left(\frac{x^2}{3}\right)^2 dx = \pi \int_0^1 \frac{x^4}{9} dx = \frac{\pi}{9} \times \frac{x^5}{5} \Big|_0^1 = \frac{\pi}{45}$$

$$(b) \quad \int_0^1 \left(\frac{x^2}{3} - (-3)\right)^2 \pi dx - 3^2 \pi = \frac{x^5}{45} + \frac{2x^3}{3} + 9x \Big|_0^1 = \frac{31\pi}{45} \quad \text{or} \quad \int_0^1 \left(\frac{x^2}{3} - (-3)\right)^2 \pi dx = \frac{436\pi}{45}$$

$$8. \quad P_d = -0.3x + 10 \quad \dots\dots \quad \text{Demand}$$

$$P_s = 0.1x + 2 \quad \dots\dots \quad \text{Supply}$$

$$P_d = P_s$$

$$\Rightarrow -0.3x + 10 = 0.1x + 2$$

$$\Rightarrow x = 20$$

$$\text{then } P_d = P_s = 4$$

$$\text{consumer surpluses} = \int_0^{20} (\text{demand function} - \text{price}) dx$$

$$= \int_0^{20} (-0.3x + 10 - 4) dx$$

$$= -0.15x^2 + 6x \Big|_0^{20}$$

$$= 60$$

$$\text{producer surpluses} = \int_0^{20} (\text{price} - \text{supply function}) dx$$

$$= \int_0^{20} 4 - (0.1x + 2) dx$$

$$= 2x - 0.05x^2 \Big|_0^{20}$$

$$= 20$$