Calculus Final Exam Solution

Exam Set:B

1. (a) $\int_0^1 x(x+1)^{10}dx = \int_0^1 (x+1-1)(x+1)^{10}dx$

$= \int_0^1 (x+1)^{11} - (x+1)^{10}d(x+1)$

$= \frac{1}{12}(x+1)^{12} - \frac{1}{11}(x+1)^{11}|_0^1$

≈ 155.159

(b) $\int_0^1 (x+3)\sqrt{2-x}dx = -\int_0^1 (-x-3)\sqrt{2-x}dx$

$= -\int_0^1 (2-x-5)\sqrt{2-x}dx$

$= -\int_0^1 (2-x)^{\frac{3}{2}} - 5(2-x)^{\frac{1}{2}}dx$

$= \int_0^1 (2-x)^{\frac{3}{2}} - 5(2-x)^{\frac{1}{2}}d(2-x)$

$= \frac{2}{5}(2-x)^{\frac{5}{2}} - \frac{10}{3}(2-x)^{\frac{3}{2}}|_0^1$

$= -\frac{44}{15} + \frac{76\sqrt{2}}{15} \approx 4.2320$

(c) $\int_0^1 \frac{e^{2x}}{e^{2x}+1}dx = \frac{1}{2}\int_0^1 \frac{1}{e^{2x}+1}d(e^{2x}+1)$

$= \frac{1}{2}\ln(e^{2x}) + 1|_0^1$

$= \frac{1}{2}[\ln(e^2+1) - \ln 2] \approx 0.7169$

(d) $\int_0^4 |2x-1|dx = \int_{\frac{1}{2}}^4 (2x-1)dx + \int_0^{\frac{1}{2}}(1-2x)dx$

$= x^2 - x|_{\frac{1}{2}}^4 + x - x^2|_0^{\frac{1}{2}}$

$= \frac{25}{2}$

2. $y = (x^2+1)^{2x+3}$

$\Rightarrow \ln y = (2x+3)\ln(x^2+1)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 2\ln(x^2+1) + (2x+3)\frac{2x}{x^2+1}$

$\Rightarrow \frac{dy}{dx} = 2[\ln(x^2+1)](x^2+1)^{2x+3} + (4x^2+6x)(x^2+1)^{2x+2}$

3. (1) $\frac{dR}{dx} = 50 - 0.02x + \frac{100}{x+1}$

$\Rightarrow R = 50x - 0.01x^2 + 100 \ln|x+1| + C$

$x = 0 \quad , \quad R = 0$

$\Rightarrow 0 = 100 \ln 1 + C$

$\Rightarrow C = 0$

$hence \quad R = 50x - 0.01x^2 + 100 \ln|x+1|$

(2) $x = 1500$

$R = 50 \times 1500 - 0.01 \times 1500^2 + 100 \times \ln(1500+1)$

$= 75000 - 22500 + 731.3887 \approx 53231.3887$

4. $\frac{dV}{dt} = 10000(t-6)$

$\Rightarrow V = 5000t^2 - 60000t$

$V(3) = -135000$

$V(0) = 0$

$V(3) - V(0) = -135000$

hence loss 135000

5. (a) $\int_0^1 \pi(\frac{x^2}{2})^2 dx = \pi \frac{x^5}{20}|_0^1 = \frac{\pi}{20}$

(b) $\int_0^1 ((\frac{x^2}{2} - (-2))^2)\pi dx - 2^2\pi = \pi\frac{x^5}{20} + \frac{2x^3}{3} + 4x|_0^1 - 4\pi = \frac{43\pi}{60}$ or $\int_0^1 ((\frac{x^2}{2} - (-2))^2)\pi dx = \frac{283\pi}{60}$

6. *Let* $\quad f(x) = \frac{5}{x^3+1}$

$$\begin{aligned} \int_0^2 \frac{5}{x^3+1} dx &\approx \frac{1}{2}[f(\frac{1}{4}) + f(\frac{3}{4}) + f(\frac{5}{4}) + f(\frac{7}{4})] \\ &\approx 5.4596 \end{aligned}$$

7. $\int_1^2 (x-1)-(x-1)^3 dx + \int_0^1 (x-1)^3-(x-1)dx$

$= \int_1^2 (x-1-x^3+3x^2-3x+1)dx + \int_0^1 (x^3-3x^2+3x-1-x+1)dx$

$= \int_1^2 (-x^3+3x^2-2x)dx + \int_0^1 (x^3-3x^2+2x)dx$

$= \frac{1}{2}$

8. $P_d = -0.3x + 12 \;\; ...... \quad Demand$

$P_s = 0.2x + 2 \;\; ...... \quad Supply$

$P_d = P_s$

$\Rightarrow -0.3x + 12 = 0.2x + 2$

$\Rightarrow \quad x = 20$

$then \;\; P_d = P_s = 6$

$consumer \;\; surpluses = \int_0^{20} (demand \;\; function - price)dx$

$= \int_0^{20} (-0.3x + 12 - 6)dx$

$= -0.15x^2 + 6x|_0^{20}$

$= 60$

$producer \;\; surpluses = \int_0^{20} (price - supply \;\; function)dx$

$= \int_0^{20} 6 - (0.2x + 2)dx$

$= 4x - 0.1x^2|_0^{20}$

$= 40$