Put down your arguments next to the answers of the multiple choice problems for partial credits.

8% (1) Students in a calculus class were given an exam and then retested monthly with equivalent exams. The average score S (on a 100-point scale) for the class can be modeled by

$$S = 68 - 12 \cdot \ln(t+1), \qquad 0 \le t \le 11$$

where $t+1$ is the time in months (so $t=0$ is the first time of test). After how many months was the average score below 45 for the first time?

Solution. Note that S is decreasing. Rewrite $45 = 68 - 12 \cdot \ln(t+1)$ as $\ln(t+1) = \frac{45-68}{-12} = \frac{23}{12}$. Solve the last equation to get $t = e^{23/12} - 1 = 5.798259795$. So, when $t = 6$, i.e. after 7 months, the average score fall below 45 for the first time.

8% (2) After t years, the value V of a car purchased for \$80,000 is

$$V = 80,000 \cdot (0.75)^{kt}.$$

It is known that immediately after 5 years, the car worths \$50,000. If immediately after s years, the car worths between \$25,000 and \$26,000, then s is

Solution. First, solve k by the equation $50,000 = 80,000 \cdot (0.75)^{5k}$: Rewriting, we have $\frac{5}{8} = (0.75)^{5k}$, and so $k = \frac{\ln(5/8)}{5 \cdot \ln 0.75} = 0.3267521158$. Now, we note that V is a decreasing function since $0.75 < 1$. Rewrite $26,000 = 80,000 \cdot (0.75)^{0.3267521158 \cdot t}$ as $(0.75)^{0.3267521158 \cdot t} = \frac{26}{80}$ and solve for t to get $t = 11.95661083$. Therefore, immediately after 12 years, the value of the car will drop below 26,000 for the first time.

8% (3) You want to deposit in a saving account that is *compounded monthly* at an annual interest rate 2.76%. If you wish to have \$100,000 in your account after 12 years, how much do you need to put into the account now?

Solution. Let P be the amount to be put in the account initially. Then we have to solve for t in the following equation: $100,000 = P \cdot (1 + 0.0276/12)^{12 \cdot 12} = P \cdot (1.0023)^1 44 = 1.392108777 \cdot P$. Thus, $P = 100,000/1.392108777 = 71833.47$.

9% (4) For the following questions, use the data given in the graph (where 0 corresponds to they year 1990). (Note. A million = 10^6, and a billion = 10^9.)

(a) The percent increase in revenue (or sales) between 1995 and 1998 was about: $(1600 - 1020)/1020 \approx 0.5686$

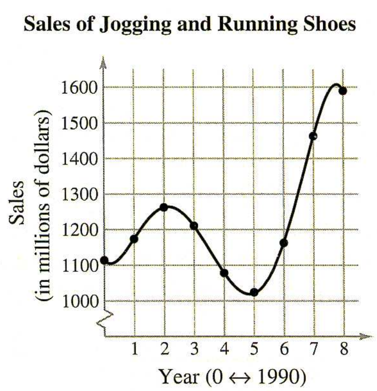
(b) Which of the following statements about the sales of jogging and running shoes can be concluded from the graph?

(I) In 1993, the sales were greater than \$1.2 billion. CORRECT

(II) Between 1995 and 1997, the average increase in sales per year was about \$55 million. WRONG
$((1460 - 1020)/2 = 220$ (million))

(III) The greatest one-year increase in sales occurred between 1996 and 1997. CORRECT

(c) Let $f(x)$ represent the revenue (sales) function. Then most likely $f'(5) = 1$



8% (5) The demand and cost functions for a product are

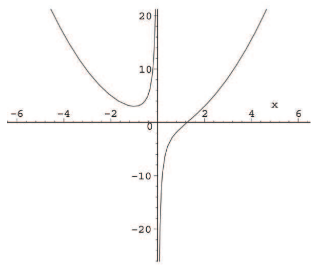
$$p = 6000 - 45x \quad \text{and} \quad C = 2x^2 + 28800$$

where p is the price per unit, x is the number of units, and C is the total cost.

What will the revenue be when the level of production minimizes the average cost per unit?

Solution. The average cost per unit is give by $A = C/x = 2x + \frac{28800}{x}$. Solving $0 = A' = 2 - \frac{28800}{x^2}$ for x, we get $x = 120$. Since $A'' < 0$, $A(120)$ is a local minimum. For $x = 120$, the revenue is $120 \cdot p(120) = 120 \cdot (6000 - 45 \cdot 120) = 72000$.

8% (6) The graph of $f(x) = x^2 - \dfrac{2}{x}$ is



14% (7) Find the indefinite integrals. Write details for full mark.

(a) $\displaystyle\int \frac{(\sqrt{x}+1)^2}{\sqrt{x}}\,dx$

$= \displaystyle\int \left(\sqrt{x} + 2 + \frac{1}{\sqrt{x}}\right) dx$
$= \frac{2}{3}x^{\frac{3}{2}} + 2x + 2\sqrt{x}$

(b) $\displaystyle\int (x^4 - 2x)^3(2x^3 - 1)\,dx$

$= \frac{1}{2}\displaystyle\int u^3\,du = \frac{1}{2} \cdot \frac{1}{4}u^4$
$= \frac{1}{8}(x^4 - 2x)$

21% (8) Find the derivative of the function. Write details for full mark.

(a) $y = \dfrac{e^x}{x^2}$

$y' = \frac{e^x \cdot x^2 - 2x \cdot e^x}{x^4} = \frac{e^x(x-2)}{x^3}$

(b) $y = \ln\left(\dfrac{x+1}{x-1}\right)^2$

$y = 2 \cdot (\ln(x+1) - \ln(x-1))$
$y' = 2 \cdot \left(\frac{1}{x+1} - \frac{1}{x-1}\right)$

(c) $y = \log_3(e^{2x}\sqrt{e^2x - 1})$

$y = \log_3 e^{2x} + \frac{1}{2}\log_3(e^2x - 1)$
$= \frac{1}{\ln 3} \cdot 2x + \frac{1}{2 \cdot \ln 3}\ln(e^2x - 1)$
$y' = \frac{2}{\ln 3} + \frac{e^2}{2 \cdot \ln 3} \cdot \frac{1}{e^2x - 1}$

8% (9) Let $g(x) = \frac{e^{-x}}{1 - 2e^x}$. Find the local maxima and local minima of the graph of $g(x)$ if they exist. You need to explain why they are local maxima or minima.

Solution. $g'(x) = -\frac{e^{-x} - 4}{(-1 + 2 \cdot e^x)^2}$; $g'(x) = 0$ when $x = -\ln(4)$. Since $g'(x) > 0$ if $x > -\ln(4)$ and $g'(x) < 0$ if $x < -\ln(4)$, $g(-\ln(4)) = 8$ is a local minimum.

8% (10) The marginal cost for producing x units of a product is modeled by $\dfrac{dC}{dx} = 128 - 0.03x$. Suppose that it costs \$12500 to produce 100 unit.

(a) Find the cost function.

Solution. $C = \displaystyle\int (128 - 0.03x)\,dx = 128x - 0.015x^2 + K$; $12500 = C(100) = 128 \cdot 100 - 0.015 \cdot 10000 + K = 12650 + K$; $K = 150$.

(b) Suppose that the demand functions is given by $p = 1280 - 5x$. Find the profit when the production level is 200.

Solution. profit $= 200 \cdot p(200) - C(200) = 200(1280 - 5 \cdot 200) - (128 \cdot 200 - 0.015 \cdot 40000 + 150) = 30850$