Calculus Midterm #2 (Form D)

Fall 2003

(1) When the tangent line exists at an inflection point, does it definitely cross the graph of the function? Why? 10%

(2) Let $f(x) = \sqrt{|x-1|}$. Discuss whether f is continuous or differentiable at $x = 1$? 10%

(3) Find the slope of the tangent at $(-2, 2)$ of a curve $y^2 = \dfrac{20 - x^2}{2x}$. 10%

(4) The demand equation is given by $p = \sqrt[3]{9 - x^3}$ where p is the unit price at which x units of the product are demanded. Define the price elasticity of demand as $\eta = \frac{p/x}{dp/dx}$.

(i) Is the demand elastic ($|\eta| > 1$), inelastic($|\eta| < 1$), or of unit elastic ($|\eta| = 1$) at $x = 1$? Give an economic interpretation for your answer. 10%

(ii) Find the expression for the total revenue and compute the values of x^* and p^* that maximize the total revenue. 10%

(iii) Show that the demand at x^* is of unit elastic. Moreover, on the interval $(x^*, 3)$ the demand is inelastic and the total revenue is decreasing. 10%

(5) The concentration C (in milligrams per milliliter) of a drug in a patient's bloodstream t hours after injection into muscle tissue is modeled by

$$C = \frac{3t}{27 + t^3}.$$

(i) Find the change in the concentration when t changes from $t = 1.5$ to $t = 2$. 5%

(ii) Use differentials to approximate the change. 5%

(6) Let

$$f(x) = \frac{1}{x^2 + 1}.$$

(i) Find all critical numbers, relative extrema and points of inflection. 10%

(ii) Determine (with reasons) whether f has vertical or horizontal asymptotes. 10%

(iii) Sketch the graph of f. 10%