

Calculus Midterm #2 (Form B)

Fall 2003

- (1) When the tangent line exists at an inflection point, does it definitely cross the graph of the function? Why? 10%

- (2) Find the slope of the tangent at $(2, 2)$ of a curve $y^2 = \frac{20 - x^2}{2x}$. 10%

- (3) Let $f(x) = \sqrt{|1 - x|}$. Discuss whether f is continuous or differentiable at $x = 1$? 10%

- (4) The concentration C (in milligrams per milliliter) of a drug in a patient's bloodstream t hours after injection into muscle tissue is modeled by

$$C = \frac{3t}{27 + t^3}.$$

- (i) Find the change in the concentration when t changes from $t = 1.5$ to $t = 2$. 5%
- (ii) Use differentials to approximate the change. 5%
- (5) The demand equation is given by $p = \sqrt[3]{9 - x^3}$ where p is the unit price at which x units of the product are demanded. Define the price elasticity of demand as $\eta = \frac{p/x}{dp/dx}$.
- (i) Is the demand elastic ($|\eta| > 1$), inelastic ($|\eta| < 1$), or of unit elastic ($|\eta| = 1$) at $x = 2$? Give an economic interpretation for your answer. 10%
- (ii) Find the expression for the total revenue and compute the values of x^* and p^* that maximize the total revenue. 10%
- (iii) Show that the demand at x^* is of unit elastic. Moreover, on the interval $(0, x^*)$ the demand is elastic and the total revenue is increasing. 10%

- (6) Let

$$f(x) = \frac{3}{x^2 + 2}.$$

- (i) Find all critical numbers, relative extrema and points of inflection. 10%
- (ii) Determine (with reasons) whether f has vertical or horizontal asymptotes. 10%
- (iii) Sketch the graph of f . 10%