Calculus Midterm #2 (Form A)

Fall 2003

(1) The concentration C (in milligrams per milliliter) of a drug in a patient's bloodstream t hours after injection into muscle tissue is modeled by

$$C = \frac{3t}{27 + t^3}.$$

(i) Find the change in the concentration when t changes from $t = 1$ to $t = 1.5$. 5%

(ii) Use differentials to approximate the change. 5%

(2) Let $f(x) = \sqrt{|x - 2|}$. Discuss whether f is continuous or differentiable at $x = 2$? 10%

(3) When the tangent line exists at an inflection point, does it definitely cross the graph of the function? Why? 10%

(4) Find the slope of the tangent at $(2, -2)$ of a curve $y^2 = \dfrac{x^3}{4 - x}$. 10%

(5) The demand equation is given by $p = \sqrt[3]{9 - x^3}$ where p is the unit price at which x units of the product are demanded. Define the price elasticity of demand as $\eta = \frac{p/x}{dp/dx}$.

(i) Is the demand elastic ($|\eta| > 1$), inelastic($|\eta| < 1$), or of unit elastic ($|\eta| = 1$) at $x = 1$? Give an economic interpretation for your answer. 10%

(ii) Find the expression for the total revenue and compute the values of x^* and p^* that maximize the total revenue. 10%

(iii) Show that the demand at x^* is of unit elastic. Moreover, on the interval $(0, x^*)$ the demand is elastic and the total revenue is increasing. 10%

(6) Let

$$f(x) = \frac{-3}{x^2 + 2}.$$

(i) Find all critical numbers, relative extrema and points of inflection. 10%

(ii) Determine (with reasons) whether f has vertical or horizontal asymptotes. 10%

(iii) Sketch the graph of f. 10%