Exam Set: D

No credit will be given for an answer without reasoning.

1.

    (1) [5%] Find $f'(1)$ for $f(x) = x(3x^2 + 2)$.

    (2) [5%] Find $f''(t)$ for $f(t) = -2t^2 + 3t$.

2.

    (1) [5%] Find an equation of the tangent line to the graph of the function f defined by

$$x^2 y^3 - y^2 - xy - 1 = 0$$

    at the point $(-1, 1)$.

    (2) [5%] Find the second derivative d^2y/dx^2 of the function defined implicitly by the equation $x^{2/3} + y^{2/3} = 1$.

3.

    (1) [5%] Find the constant a, b such that the function
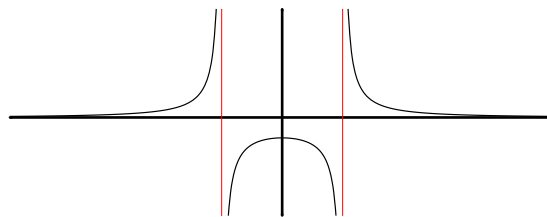
$$f(x) = \begin{cases} 2, & \text{if } x \leq -1 \\ ax + b, & \text{if } -1 < x < 3 \\ -2, & \text{if } x \geq 3 \end{cases}$$

    is continuous on the entire real line.

    (2) [5%] Find $\lim_{\Delta t \to 0} \frac{\sqrt{t+3+\Delta t} - \sqrt{t+3}}{\Delta t}$.

4.

    (1) [5%] Given the graph of $y = f(x)$ below, sketch the graph of $y = f'(x)$.

    (2) [5%] Suppose that f and g are functions that are differentiable at $x = 1$ and that $f(1) = -2$, $f'(1) = 1$, $g(1) = -2$ and $g'(1) = 3$. Find the value $h'(1)$ where $h(x) = \frac{xf(x)}{x+g(x)}$.

5. True or False? Determine whether the statement is true or false. Explain your answer.

    (1) [5%]

$$\lim_{x \to -1} \left(\frac{2x}{x+1} + \frac{2}{x+1} \right) = \lim_{x \to -1} \frac{2x}{x+1} + \lim_{x \to -1} \frac{2}{x+1}$$

    (2) [5%] If a function f is continuous at $x = a$, then f is differentiable at $x = a$.

6.

(1) [5%] Find the marginal cost for producing x units when the cost function is $C = 90(8 + 3\sqrt{x})$.

(2) [5%] A point is moving along the graph of $y = x^2$ so that dx/dt is 2 centimeters per minute. Find dy/dt for $x = 2$.

7.

(1) [5%] According to the Doyle Log Rule, the volume V of a log of length L (in feet) and diameter D (in inches) at the small end is

$$V = \left(\frac{D - 4}{4}\right)^2 L.$$

Find the rate at which the volume is changing with respect to D for a 12-foot-long log whose smallest diameter is 16 inches.

(2) [5%] The temperature T (in degree F) of food placed in a freezer can be modelled by

$$T = \frac{1300}{t^2 + 2t + 25}$$

where t is the time (in hours). Find the rate of change of T when $t = 3$.

8. The position function of a particle is given by

$$s = t^3 - 6t^2 - 10t, \qquad t \geq 0$$

where s is measured in meters and t is measured in seconds.

(1) [5%] When does the particle reach a velocity of 5 m/sec?

(2) [5%] When is the acceleration 0?

9. [10%] The management of Cruise World, operator of Caribbean luxury cruises, have constructed the following model, which gives the percentage of young adult passengers in year t:

$$p = 50\left(\frac{t^2 + 2t + 4}{t^2 + 4t + 8}\right) \qquad (0 \leq t \leq 5).$$

Young adults normally pick shorter cruises and generally spend less on their passage. The following model gives the average amount of money R (in dollars) spent per passenger on a cruise when the percentage of young adult is p:

$$R = 1000\left(\frac{p + 4}{p + 2}\right)$$

Find the rate at which the price of average will be changing 2 years from now.

10. [10%] A 20 feet ladder leaning against a wall begins to slide. How fast is the top of the ladder sliding down the wall at the instant of time when the bottom of the ladder is 16 feet

from the wall and sliding away from the wall at the rate of 5 foot/sec?