CALCULUS MIDTERM 1

Exam Set: C

No credit will be given for an answer without reasoning.

1.

(1) [5%] Find the constant c, d such that the function

$$f(x) = \begin{cases} 4, & \text{if } x \leq -1 \\ cx + d, & \text{if } -1 < x < 3 \\ -4, & \text{if } x \geq 3 \end{cases}$$

is continuous on the entire real line.

(2) [5%] Find $\lim_{\Delta x \to 0} \frac{\sqrt{x-1+\Delta x} - \sqrt{x-1}}{\Delta x}$.

2.

(1) [5%] Find $f'(1)$ for $f(x) = (x+1)(x^2+1)$.

(2) [5%] Find $f''(t)$ for $f(t) = -t^2 + 4t + 2$.

3.

(1) [5%] Find an equation of the tangent line to the graph of the function f defined by

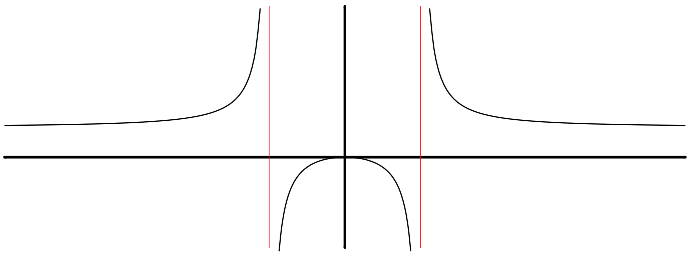
$$x^2y^3 - y^2 + xy - 1 = 0$$

at the point $(1, 1)$.

(2) [5%] Find the second derivative d^2y/dx^2 of the function defined implicitly by the equation $x^{1/3} + y^{1/3} = 1$.

4.

(1) [5%] Given the graph of $y = f(x)$ below, sketch the graph of $y = f'(x)$.



(2) [5%] Suppose that f and g are functions that are differentiable at $x = 1$ and that $f(1) = 2$, $f'(1) = -1$, $g(1) = -2$ and $g'(1) = 3$. Find the value $h'(1)$ where $h(x) = \frac{xf(x)}{x - g(x)}$.

5. True or False? Determine whether the statement is true or false. Explain your answer.

(1) [5%]

$$\lim_{x \to 1} \left(\frac{2x}{x-1} - \frac{2}{x-1} \right) = \lim_{x \to 1} \frac{2x}{x-1} - \lim_{x \to 1} \frac{2}{x-1}$$

(2) [5%] If a function f is not continuous at $x = a$, then f is not differentiable at $x = a$.

6.

(1) [5%] Find the marginal cost for producing x units when the cost function is $C = 100(9 + 2\sqrt{x})$.
(2) [5%] A point is moving along the graph of $y = x^2$ so that dx/dt is 2 centimeters per minute. Find dy/dt for $x = 3$.

7. The position function of a particle is given by

$$s = t^3 - 4.5t^2 - 7t, \qquad t \geq 0$$

where s is measured in meters and t is measured in seconds.

(1) [5%] When does the particle reach a velocity of 5 m/sec?
(2) [5%] When is the acceleration 0?

8.

(1) [5%] According to the Doyle Log Rule, the volume V of a log of length L (in feet) and diameter D (in inches) at the small end is

$$V = \left(\frac{D-4}{4}\right)^2 L.$$

Find the rate at which the volume is changing with respect to D for a 12-foot-long log whose smallest diameter is 8 inches.

(2) [5%] The temperature T (in degree F) of food placed in a freezer can be modelled by

$$T = \frac{1300}{t^2 + 2t + 25}$$

where t is the time (in hours). Find the rate of change of T when $t = 5$.

9. [10%] The quantity demanded per month, x, of a certain make of personal computer (PC) is related to the average unit price, p (in dollars), of PCs by the equation

$$x = \frac{100}{9}\sqrt{810,000 - p^2}.$$

It is estimated that t months from now, the average price of a PC will be given by

$$p = \frac{400}{1 + \frac{1}{8}\sqrt{t}} + 200 \qquad (0 \leq t \leq 60)$$

dollars. Find the rate at which the quantity demanded per month of the PCs will be changing 16 month from now?

10. [10%] A 20 feet ladder leaning against a wall begins to slide. How fast is the top of the ladder sliding down the wall at the instant of time when the bottom of the ladder is 12 feet

from the wall and sliding away from the wall at the rate of 5 foot/sec?