CALCULUS FINAL EXAM— PAPER B **JANUARY 8, 2004**

請在答案卷上清楚標明題目卷之種類 **(B)** 並寫下姓名

No credit will be given for an answer without reasoning.

1. Evaluate

(a) [8%] $\int_0^1 x(x+1)^{10}\,dx$

(b) [8%] $\int_0^1 (x+3)\sqrt{2-x}\,dx$

(c) [8%] $\int_0^1 \frac{e^{2x}}{e^{2x}+1}\,dx$

(d) [8%] $\int_0^4 |2x-1|\,dx$

2. [8%] Find the derivative of $y=(x^2+1)^{2x+3}$.

3. The marginal revenue (邊際營業額) for the sale of a product can be modelled by $\frac{dR}{dx}=50-0.02x+\frac{100}{x+1}$, where x is the quantity demanded.

(1) [6%] Find the revenue function R.

(2) [4%] Find the revenue when 1500 units are sold.

4. [8%] A company purchases a new machine (購買了一台新機器) for which the rate of depreciation (折舊速率) can be modelled by

$$\frac{dV}{dt}=10{,}000\,(t-6),\ 0\le t\le 5$$

where V is the value of the machine after t years. Set up and evaluate the definite integral that yields the total loss of value of the machine over the first 3 years (給出前三年機器價值的總損失).

5. Find the volume of the solid obtained by revolving the curve $y=\frac{x^2}{2}$ from $(0,0)$ to $(1,\frac{1}{2})$

(a) [6%] about x-axis.

(b) [6%] about the line $y=-2$.

6. [10%] Use the Midpoint Rule with $n=4$ to approximate (逼近) $\int_0^2 \frac{5}{x^3+1}\,dx$.

7. [10%] Find the area of the region bounded by the graphs of $f(x)=(x-1)^3$, and $g(x)=x-1$ from $x=0$ to $x=2$.

8. [10%] The demand (需求) and supply (供應) functions for a product are

$$\text{Demand: } p=-0.3x+12 \text{ and Supply: } p=0.2x+2$$

where x is the number of units (in millions). Find the consumer (消費者) and producer (製造者) surpluses (盈餘) for this product.

Hint: Let (x_0,p_0) be the point at which a demand function and a supply function intersect. Economists (經濟學者) call the area of the region bounded by the graph of the demand function, the horizontal line (水平線) $p=p_0$, and the vertical line (垂直線) $x=0$ the **consumer surplus**. Similarly, the area of the region bounded by the graph of the supply function, the horizontal line $p=p_0$, and the vertical line $x=0$ is called the **producer surplus**.