

請在答案卷上清楚標明題目卷之種類 (A) 並寫下姓名

No credit will be given for an answer without reasoning.

1. Evaluate

(a) [8%] $\int_0^4 |2x - 1| dx$

(b) [8%] $\int_0^1 x(x+1)^{10} dx$

(c) [8%] $\int_0^1 (x+3)\sqrt{2-x} dx$

(d) [8%] $\int_0^1 \frac{e^{2x}}{e^{2x}+1} dx$

2. [8%] Find the derivative of $y = (x^2 + 1)^{5x+2}$.

3. [8%] A company purchases a new machine (購買了一台新機器) for which the rate of depreciation (折舊速率) can be modelled by

$$\frac{dV}{dt} = 10,000(t-6), \quad 0 \leq t \leq 5$$

where V is the value of the machine after t years. Set up and evaluate the definite integral that yields the total loss of value of the machine over the first 3 years (給出前三年機器價值的總損失).

4. The marginal revenue (邊際營業額) for the sale of a product can be modelled by $\frac{dR}{dx} = 50 - 0.02x + \frac{100}{x+1}$, where x is the quantity demanded.

(1) [6%] Find the revenue function R .

(2) [4%] Find the revenue when 1500 units are sold.

5. Find the volume of the solid obtained by revolving the curve $y = \frac{x^2}{3}$ from $(0,0)$ to $(1, \frac{1}{3})$

(a) [6%] about x -axis.

(b) [6%] about the line $y = -3$.

6. [10%] Use the Midpoint Rule with $n = 4$ to approximate (逼近) $\int_0^2 \frac{5}{x^3+1} dx$.

7. [10%] Find the area of the region bounded by the graphs of $f(x) = (x-1)^3$, and $g(x) = x-1$ from $x = 0$ to $x = 2$.

8. [10%] The demand (需求) and supply (供應) functions for a product are

$$\text{Demand: } p = -0.2x + 12 \text{ and Supply: } p = 0.3x + 2$$

where x is the number of units (in millions). Find the consumer (消費者) and producer (製造者) surpluses (盈餘) for this product.

Hint: Let (x_0, p_0) be the point at which a demand function and a supply function intersect. Economists (經濟學者) call the area of the region bounded by the graph of the demand function, the horizontal line (水平線) $p = p_0$, and the vertical line (垂直線) $x = 0$ the **consumer surplus**. Similarly, the area of the region bounded by the graph of the supply function, the horizontal line $p = p_0$, and the vertical line $x = 0$ is called the **producer surplus**.