Calculus Examination2 Solution

TEST D

1. $f_x = e^{\cos y}$

   $f_y = -e^{\cos y} x \sin y$

   $f_{xx} = 0$

   $f_{xy} = f_{yx} = -e^{\cos y} \sin y$

   $f_{yy} = -e^{\cos y} x \cos y + e^{\cos y} x \sin^2 y$

2. $f(x) = \frac{\cos x}{1+3\sin^2 x}$

   $f'(x) = -\frac{\sin x}{1+3\sin^2 x} - \frac{6\cos^2 x \sin x}{(1+3\sin^2 x)^2} = \frac{-\sin x(1+3\sin^2 x+6\cos^2 x)}{(1+3\sin^2 x)^2} = \frac{-\sin x(4+3\cos^2 x)}{(1+3\sin^2 x)^2}$

   $f'(x) = 0 \Rightarrow \sin x = 0 \Rightarrow x = k\pi \quad , k = 0, \pm 1, \pm 2, \pm 3, \ldots$

   $f(0 + 2k\pi) = 1$ is the relative maximum

   $f(\pi + 2k\pi) = -1$ is the relative minimum

3. Let f(x) $= e^{3x} \tan(\frac{x}{3})$

   f'(x) $= 3e^{3x} \tan(\frac{x}{3}) + \frac{1}{3} e^{3x} \sec^2(\frac{x}{3})$

   f'(0) $= \frac{1}{3}$

   $\Rightarrow y = \frac{1}{3}x$ is the tangent line at point (0,0)

4. $\int_0^4 \int_0^4 (xy)^2 \, dx \, dy = \int_0^4 \frac{x^3}{3} y^2 \mid_0^4 dy = \frac{64}{3} \cdot \frac{y^3}{3} \mid_0^4 = \frac{4096}{9}$

5. (a) $a = \frac{4\times(100\times 35+150\times 44+200\times 50+250\times 56)-(100+150+200+250)\times(35+44+50+56)}{(100^2+150^2+200^2+250^2)-(100+150+200+250)^2} = 0.138$

   $b = \frac{1}{4} \cdot (35 + 44 + 50 + 56) - a(100 + 150 + 200 + 250) = 22.1$

   hence the least square regression line is $y = 0.138x + 22.1$

   (b) $0.138 \times 160 + 22.1 = 44.18$

6. (a) $\int_0^{12}\ 10+5\sin\frac{\pi(t-8)}{6}\ dt = 10t - \frac{30}{\pi}\cos\frac{\pi(t-8)}{6}\ |_0^{12} = 120 - \frac{30}{\pi}(-\frac{1}{2}) + \frac{30}{\pi}(-\frac{1}{2}) = 120$

(b) $\frac{1}{12-6}\int_6^{12}\ 10+5\sin\frac{\pi(t-8)}{6}\ dt$

$= \frac{1}{6}(10t - \frac{30}{\pi}\cos\frac{\pi(t-8)}{6})\ |_6^{12}$

$= 10 - \frac{5}{\pi}(-\frac{1}{2}) + \frac{5}{\pi}(\frac{1}{2})$

$= 10 + \frac{5}{\pi}$

7. (a) $\int\ x\sin x\ dx = \int\ -x\ d\cos x = -x\cos x + \int\ \cos x\ dx = -x\cos x + \sin x + c$

(b) $\int_0^3\ \int_0^{x^2}\ \sqrt{1+x}\ dy\ dx$

$= \int_0^3\ y\sqrt{1+x}\ |_0^{x^2}\ dx$

$= \int_0^3\ x^2\sqrt{1+x}\ dx$

$= \int_0^3\ (x+1-1)^2\sqrt{x+1}\ d(x+1)$

$= \int_0^3\ (x+1)^{\frac{5}{2}} - 2(x+1)^{\frac{3}{2}} + (x+1)^{\frac{1}{2}}\ d(x+1)$

$= \frac{2}{7}(x+1)^{\frac{7}{2}} - \frac{4}{5}(x+1)^{\frac{5}{2}} + \frac{2}{3}(x+1)^{\frac{3}{2}}\ |_0^3$

$= \frac{1696}{105}$

(c) $\int\ \frac{3-2\sin^2(\frac{x}{2})}{2x+\sin x}\ dx = \frac{1}{2}\int\ \frac{2+\cos x}{2x+\sin x}\ dx = \frac{1}{2}\ln|2x+\sin x| + c$

(d) $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\ (2x+\cos x)^2\ dx$

$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\ 4x^2 + 4x\cos x + \cos^2 x\ dx$

$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\ 4x^2 + 4x\cos x + \frac{\cos 2x+1}{2}\ dx$

$= \frac{4}{3}x^3 + 4x\sin x + 4\cos x + \frac{1}{4}\sin 2x + \frac{x}{2}\ |_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$

$= \frac{\sqrt{3}}{4} + \frac{\pi}{6} + \frac{\pi^3}{81}$

(e) $\int_0^{\pi}\ \frac{1}{\cos x}dx$

$= \int_0^{\pi}\ \sec x\ dx$

$= \lim_{a\to\frac{\pi}{2}^-}\ \int_0^a\ \sec x\ dx + \lim_{b\to\frac{\pi}{2}^+}\ \int_b^{\pi}\ \sec x\ dx$

$= \lim_{a\to\frac{\pi}{2}^-}\ \int_0^a\ \frac{\sec x(\sec x+\tan x)}{\sec x+\tan x}\ dx + \lim_{b\to\frac{\pi}{2}^+}\ \int_b^{\pi}\ \frac{\sec x(\sec x+\tan x)}{\sec x+\tan x}\ dx$

$= \lim_{a\to\frac{\pi}{2}^-} \int_0^a \frac{\sec^2 x+\sec x\tan x}{\sec x+\tan x}\, dx + \lim_{b\to\frac{\pi}{2}^+} \int_b^\pi \frac{\sec^2 x+\sec x\tan x}{\sec x+\tan x}\, dx$

$= \lim_{a\to\frac{\pi}{2}^-} \ln|\sec x+\tan x|\ |_0^a + \lim_{b\to\frac{\pi}{2}^+} \ln|\sec x+\tan x|\ |_b^\pi$

$= \lim_{a\to\frac{\pi}{2}^-} \ln|\sec a+\tan a| - \lim_{b\to\frac{\pi}{2}^+} \ln|\sec b+\tan b|$

since $\lim_{a\to\frac{\pi}{2}^-} \ln|\sec a+\tan a|$ and $\lim_{b\to\frac{\pi}{2}^+} \ln|\sec b+\tan b|$ are divergent

hence $\int_0^\pi \frac{1}{\cos x}\, dx$ *is divergent*

8. (a) $\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x+1)(x-1)}{x-1} = \lim_{x\to 1} x+1 = 2$

   (b) $\lim_{x\to\infty} \frac{x^3-x^2+4x-4}{x^3-6x^2+5x} = \lim_{x\to\infty} \frac{1-\frac{1}{x}+\frac{4}{x^2}-\frac{4}{x^3}}{1-\frac{6}{x}+\frac{5}{x^2}} = 1$

   (c) $\lim_{x\to 0} \frac{\sin(2x)}{5x}$ ($(\frac{0}{0})$ apply l'Hopital's rule)

   $= \lim_{x\to 0} \frac{2\cos(2x)}{5}$

   $= \frac{2}{5}$

   (d) $\lim_{x\to 0^+} \frac{(\ln x)^4}{x} = \infty$