MIDTERM 1 FOR CALCULUS

Time: 08:10–10:00, Thursday, Apr. 17, 2003
Instructor: Shu-Yen Pan

No credit will be given for an answer without reasoning.

1. (1) [5%] Find the radian measure of $750°$.

(2) [5%] We know that $\frac{d\sin t}{dt} = \cos t$ and $\frac{d\cos t}{dt} = -\sin t$. Use quotient rule to check that $\frac{d\tan t}{dt} = \sec^2 t$.

2. (1) [5%] For $f = e^{2x^2+3y^2+4z^2}$, find $f_y(1,-1,1)$.

(2) [5%] Rewrite the integral $\int_0^2 \int_{y^2}^4 f(x,y)\,dx\,dy$ so that x is the outer variable (i.e., change the order of the integration)

3. (1) [5%] Find the least square line for the points: $(-2,12)$, $(0,10)$, $(2,6)$, $(4,0)$, and $(6,-3)$.

(2) [5%] A rectangle is measured to have length x and width y, but each measurement may be in error by 1%. Estimate the percentage error in calculating the area.

4. (1) [5%] Find the limit $\lim_{x\to 0} \frac{\sin x}{e^x - 1}$.

(2) [5%] Evaluate the integral $\int_0^2 \frac{1}{x^2+4}\,dx$.

5. (1) [5%] Evaluate the integral $\int \cot t\,dt$.

(2) [5%] Evaluate the integral $\int t \sin t\,dt$.

6. Find the relative extreme values of the function $f(x,y) = 16xy - x^4 - 2y^2$. (You have to use D-test to characterize the critical points).

7. Find the volume of the solid bounded by the graphs of $z = x + y$, $z = 0$, $x = 0$, $x = 3$, $y = x$, and $y = 0$.

8. A boatyard estimates that sales during week x of the year will be

$$S(x) = 25 - 20\cos\frac{\pi x}{26}$$

thousand dollars, where $x = 0$ corresponds to the beginning of the year.

(1) Graph the sales function on the interval $[0, 52]$.
(2) Find the total sales during the first half of the year.

9. A boatyard builds 18-foot and 22-foot sailboats. Each 18-foot boat costs \$ 3000 to build, each 22-foot boat costs \$ 5000 to build, and the company's fixed costs are \$ 6000. The price function for the 18-foot boats is $p(x) = 7000 - 20x$, and that for the 22-foot boat is $q(y) = 8000 - 30y$ (both in dollars), where x and y are the numbers of 18-foot and 22-foot boats, respectively.

(1) Find the company's cost function $C(x,y)$.
(2) Find the company's revenue function $R(x,y)$.
(3) Find the company's profit function $P(x,y)$.
(4) Find the quantities and prices that maximize profit. Also find the maximum profit.

10. An open-top box with a square base and two perpendicular dividers, as shown in the diagram, is to have a volume of 288 cubic inches. Use Lagrange multipliers to find the dimensions that require the least amount of material.