MIDTERM 1 FOR CALCULUS

Time: 13:10–15:00, Monday, Nov 26, 2001
Instructor: Shu-Yen Pan

No calculator is allowed. No credit will be given for an answer without reasoning.

1. (1) [5%] Find the limit $\lim_{x\to\infty} \frac{-x^3+x^2+1}{100x^2-1}$.

(2) [5%] Find the limit $\lim_{s\to 16} \frac{s-16}{4-\sqrt{s}}$.

2. (1) [5%] Find an equation of the tangent line to the curve $y = \frac{x}{\sqrt{2-x^2}}$ at the point $(1,1)$.

(2) [5%] Find $\frac{dp}{dt}$ if $p = (2t-5)^3(8t^2-5)^{-3}$.

3. (1) [5%] Given the graph of $y = f(x)$ below, sketch the graph of $y = f'(x)$.

(2) [5%] Suppose that u and v are differentiable functions and that $w = u \circ v$ and $u(0) = 1$, $v(0) = 2$, $u'(0) = 3$, $v'(0) = 4$, $u'(1) = 5$, $v'(1) = 6$, $u'(2) = 7$, $v'(2) = 8$. Find $w'(0)$.

4. [5%] Use differential to estimate $\sqrt[3]{0.97}$.

5. [5%] A study estimated that the number of housing starts per year over the next five years will be

$$N(r) = \frac{7}{1+0.02r^2}$$

million units, where r (percent) is the mortgage rate. Suppose that the mortgage rate over the next t years is

$$r(t) = \frac{10t+150}{t+10} \qquad (0 \le t \le 5)$$

percent per year. Find an expression for the number of housing starts per year as a function of t (years from now).

6. [10%] Find the first and second derivatives (i.e., $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$) of the function defined implicitly by the equation $x^3 + y^3 = 28$.

7. [10%] Use the first derivative test to find local extreme values of the function $f(r) = \frac{r}{r^2+1}$.

8. [10%] The demand for motorcycle tires imported by a company is $40,000$ per year and may be assumed to uniform throughout the year. The cost of ordering a shipment of tires is \$400, and the cost of storing each tire for a year is \$2. Determine how many tires should be each shipment if the ordering and storage costs are to be minimized. (Assume that each shipment arrives just as the previous one has been sold.)

9. [20%] Use the guidelines in the textbook to sketch the graph of the function $y = \frac{x^2}{4-x^2}$.

10. [10%] In calm waters, the oil spilling from the ruptured hull of a grounded tanker spreads in all directions. Assuming that the area polluted is a circle and its radius is increasing at a rate of 2 feet per second, determine how fast the area is increasing when the radius of the circle is 40 feet.