

FINAL FOR CALCULUS

Instructor: Shu-Yen Pan

No credit will be given for an answer without reasoning.

1. [10%]

- (i) Find the derivative of $f(x) = \frac{\sin x}{x}$.
- (ii) Evaluate $\int_0^\pi (\sin x + \cos x) dx$.

2. [10%] Find the Taylor series of the function $f(x) = e^{1+x}$ at $x = 1$. And also find the interval of convergence.

3. [10%] Sketch the graph of the function $y = f(x) = \cos^2 x$ on the interval $[0, \pi]$ by obtaining the following information:

- (i) the intervals where f is increasing and where it is decreasing
- (ii) the relative extrema of f
- (iii) the concavity of f
- (iv) the inflection points of f

4. [10%]

- (i) The n th term of a sequence is given by $a_n = \frac{2n^2+1}{3n^2-1}$. Determine whether the sequence converges or diverges. If the sequence converges, find its limit.
- (ii) Find the sum of the geometric series $\sum_{n=1}^{\infty} (\frac{1}{e})^n$ if it converges.

5. [10%] Suppose X is a normal random variable with $\mu = 45$ and $\sigma = 4$. Find the following values by using the table appended in the exam.

- (i) $P(X \leq 50)$
- (ii) $P(40 \leq X \leq 50)$

6. [10%] Find the mean, variance and standard deviation of the random variable x associated with the probability density function $f(x) = \frac{4}{x^5}$ over the interval $[1, \infty)$.

7. [10%] Find the solution of the initial value problem: $y' = 3x^2e^{-y}$ and $y(0) = 1$.

8. [10%] Estimate the value $\sqrt[3]{6}$ by using three iterations of the Newton-Raphson method with the initial guess $x_0 = 2$ on the function $f(x) = x^3 - 6$.

9. [10%]

- (i) Evaluate $\int_1^4 \ln x dx$.
- (ii) Find $f_x(1, -2)$ for $f(x, y) = \frac{x+y}{x-y}$.

10. [10%] Find the maximum and minimum values of the function $f(x, y) = e^{xy}$ subject to the constraint $x^2 + y^2 = 8$.