FINAL FOR CALCULUS

Date: Monday, Jun 17, 2002
Instructor: Shu-Yen Pan
No credit will be given for an answer without reasoning.

1. [10%]

    (i) Find the exact value of $\sin \frac{7\pi}{6}$.

    (ii) Find the derivative of $f(t) = \sin \sqrt{t}$.

2. [10%]

    (i) The nth term of a sequence is given by $a_n = \frac{1+\sqrt{n}}{1-\sqrt{n}}$. Determine whether the sequence converges or diverges. If the sequence converges, find its limit.

    (ii) Evaluate dy/dx at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ when $\sin 2x + \cos y = 0$.

3. [10%] Evaluate the double integral $\int\int_A 2xy\, dA$ where A is the area enclosed by the curves $y = \sqrt{x}$ and $y = x^2$.

4. [10%]

    (i) Suppose that the average wage earner in a certain country saves 8% of his or her take-home pay and spends the other 92%. Estimate the impact that a proposed \$10 billion tax cut will have on the economy over the long run in terms of the additional spending generated.

    (ii) Find the area under the curve $y = \cot x$ and above the x-axis from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{2}$.

5. [10%] Suppose X is a normal random variable with $\mu = 35$ and $\sigma = 3$. Find the following values by using the table appended in the exam.

    (i) $P(X \leq 44)$

    (ii) $P(30 \leq X \leq 40)$

6. [10%] The life of a light bulb is the random variable x (in hours) associated with the probability density function $f(x) = ke^{-kx}$ over the interval $[0, \infty)$ where k is a constant. Suppose we know that the mean is 1000 hours.

    (i) Find the value k.

    (ii) Find the variance of the random variable.

7. [10%] Find the solution of the initial value problem: $y' = y^2 e^x + y^2$ and $y(0) = 1$.

8. [10%] Estimate the value $\sqrt[3]{7.8}$ by the following two methods:

    (i) Use two iterations (i.e., find x_2) of the Newton-Raphson method with the initial guess $x_0 = 2$ on the function $f(x) = x^3 - 7.8$.

    (ii) Use the second Taylor polynomial of $g(x) = \sqrt[3]{x}$ at $x = 8$.

9. [10%] Find the Taylor series of the function $f(x) = \sin 2x$ at $x = \frac{\pi}{6}$. And also find the interval of convergence.

10. [10%] A company has a monthly advertising budget of \$60,000. Their marketing department estimates that if they spend x dollars on newspaper advertising and y dollars on television advertising, then the monthly sales will be given by

$$z = f(x, y) = 90x^{1/4}y^{3/4}$$

dollars. Determine how much money the company should spend on newspaper advertising and on television advertising per month to maximize its monthly sales.