MIDTERM 1 FOR CALCULUS

Time: 8:10–9:55 AM, Friday, April 20, 2001

No calculator is allowed. No credit will be given for an answer without reasoning.

1. Suppose that $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$ and $\mathbf{b} = \mathbf{j} - 2\mathbf{k}$. Find

(1) [2%] $\mathbf{b} \times -2\mathbf{a}$,
(2) [2%] $\|\mathbf{a} + 2\mathbf{b}\|$,
(3) [2%] $\mathrm{proj}_{\mathbf{a}}\mathbf{b}$,
(4) [2%] the unit vector in the direction of $\mathbf{a}$,
(5) [2%] cos of the angle between $\mathbf{a}$ and $\mathbf{b}$.

2.

(1) [5%] Find an equation of the plane passes through the point $(1, 2, 3)$ and contains the line $x = 3t$, $y = 1 + t$, $z = 2 - t$.
(2) [5%] Find the distance between the two parallel planes: $z = x + 2y + 1$ and $3x + 6y - 3z = 4$.

3.

(1) [5%] Find the acceleration of a particle with the given position function $\mathbf{r}(t) = t^2\mathbf{i} + \ln t\mathbf{j} + t\mathbf{k}$.
(2) [5%] Find the tangential component of the acceleration vector of $\mathbf{r}(t) = e^t\mathbf{i} + \sqrt{2}t\mathbf{j} + e^{-t}\mathbf{k}$.

4. Suppose that $\mathbf{r}(t) = \frac{t^3}{3}\mathbf{i} + t^2\mathbf{j} + 2t\mathbf{k}$.

(1) [5%] Find the unit normal vector $\mathbf{N}(t)$.
(2) [5%] Find the curvature κ.

5. [10%] Find the area of the shaded region.

6.

(1) [5%] Find the length of the curve $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ for $0 \le \theta \le \pi$.
(2) [5%] Find an equation of the tangent line to the curve $x = \sin t$, $y = \sin(\sin t + t)$ at $(0, 0)$.

7.

(1) [5%] Determine the sequence $\left\{\frac{\ln(n^2)}{n}\right\}$ converges or diverges. If it converges, find the limit.
(2) [5%] How many terms of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$ do we need to add in order to find the sum with error less than 0.01.

8. [10%] Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}}(x+3)^n.$$

9. Let $f(x) = \frac{e^x - 1}{x}$.

(1) [5%] Find the power series representation of f in powers of x.
(2) [5%] Differentiate the power series in (1) and show that

$$\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1.$$

10. [10%] If $p > 1$, evaluate the expression

$$\frac{1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots}{1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \cdots}$$