MIDTERM 2 FOR CALCULUS

Time: 8:10–9:55 AM, Friday, December 15, 2000
Instructor: Shu-Yen Pan

No calculator is allowed. No credit will be given for an answer without reasoning.

1. Find

(1) [4%] $\int \frac{1}{x^2-4x+3}\,dx$.
(2) [4%] $\int \frac{1}{x^2-4x+5}\,dx$.

2. (1) [4%] Evaluate $\int_{-2}^{1} |2x+1|\,dx$.
(2) [4%] Integrate $\int \tan^4 x \sec^4 x\,dx$

3. (1) [4%] Find f' if $f(x) = (x^2)^x$.
(2) [4%] Integrate $\int \coth x\,dx$.

4. (1) [4%] Find the exact value of the expression $\sin(\cos^{-1}\frac{3}{5})$.
(2) [4%] Find the area of the region bounded by the curves $y = 20 - x^2$ and $y = x^2 - 12$.

5. [6%] Use Newton method with the specified initial approximation $x_1 = -1$ to find x_2, the second approximation to the root of the equation $x^3 + x + 1 = 0$.

6. [6%] Suppose that f is differentiable, $f(0) = 0$, $f(1) = 1$, $f'(x) > 0$ and $\int_0^1 f(x)\,dx = \frac{1}{4}$. Find the value of the integral $\int_0^1 f^{-1}(y)\,dy$.

7. [8%] Let

$$f(x) = \int_2^x \sqrt{1+t^2}\,dt.$$

Prove that $f(x)$ has an inverse and find $f'(0)$.

8. [8%] Find the integral $\int_0^2 \frac{x^2}{(x^2+4)^2}\,dx$.

9. [8%] Find the limit

$$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \sqrt{\frac{3}{n}} + \cdots + \sqrt{\frac{n}{n}}\right).$$

10. [8%] A fence 3 meters tall runs parallel to a tall building at a distance of 2 meters from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building?

11. [8%] If $x\sin x = \int_0^{x^2} f(t)\,dt$, where f is a continuous function, find $f(4)$.

12. [8%] Find the limit

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{2x}.$$

13. [8%] Let f be a function such that f' is continuous on $[a,b]$. Prove that

$$\int_a^b f(t)f'(t)\,dt = \frac{1}{2}\Big(f(b)-f(a)\Big)\Big(f(b)+f(a)\Big).$$