MIDTERM 1 FOR CALCULUS

Time: 8:10–9:55 AM, Friday, Nov 3, 2000
Instructor: Shu-Yen Pan

No calculator is allowed. No credit will be given for an answer without reasoning.

1. (1) [5%] Find the limit $\lim_{\theta \to 0} \frac{\cos \theta - 1}{\sin \theta}$.

    (2) [5%] Find the limit $\lim_{s \to 16} \frac{4 - \sqrt{s}}{s - 16}$.

2. (1) [5%] Find the limit $\lim_{x \to \infty} \sqrt{\frac{2x^2 - 1}{x + 8x^2}}$.

    (2) [5%] For what value of the constant c is the function

$$f(x) = \begin{cases} cx + 1, & \text{if } x \le 3; \\ cx^2 - 1, & \text{if } x > 3. \end{cases}$$

    continuous on $(-\infty, \infty)$?

3. (1) [5%] Given the graph of $y = f(x)$ below, sketch a graph of $y = f'(x)$.

    (2) [5%] Differentiate $f(t) = \tan(\sin t^2)$.

4. (1) [5%] Suppose that u and v are differentiable functions and that $w = u \circ v$ and $u(0) = 1$, $v(0) = 2$, $u'(0) = 3$, $u'(2) = 4$, $v'(0) = 5$, $v'(2) = 6$. Find $w'(0)$.

    (2) [5%] Show that the curves $3x^2 + 2x - 3y^2 = 1$ and $6xy + 2y = 0$ are orthogonal.

5. (1) [5%] Find an equation of the tangent line to the curve $y = \frac{|x|}{\sqrt{2 - x^2}}$ at the point $(1, 1)$.

    (2) [5%] Find $\frac{dp}{dt}$ if $p = (2t - 5)^4 (8t^2 - 5)^{-3}$.

6. [10%] Use the linear approximation of the function $f(x) = \sqrt[4]{x + 1}$ to estimate $\sqrt[4]{1.02}$.

7. [10%] Find the absolute maximum and absolute minimum values of the function $f(x) = \frac{x}{x^2 + 1}$ on the interval $[0, 2]$.

8. [20%] Use the guidelines in the textbook to sketch the graph of the function $y = \frac{x}{(x - 1)^2}$.

9. [10%] Prove that the equation $x^3 + 3x + 2 = 0$ has exactly one real root.