MIDTERM 2 FOR CALCULUS

Date: 1999, December 13, Monday, 1:10–2:50 PM
Each problem is worth 10 points.

1.

(i) Determine the domain and find the derivative of the function $g(x) = \ln|x^3 - 1|$.
(ii) Find the derivative of the function

$$f(x) = \int_x^{\cos x} \sqrt{1-t^2}\, dt.$$

2. Compute

(i) $\int \frac{x+1}{x^2}\, dx$
(ii) $\int e^{\ln x}\, dx$

3. Compute

(i) $\int_0^{\pi/2} \sin^3 x \cos x\, dx$
(ii) $\int \frac{1}{\sqrt{x}+x}\, dx$

4. Two hallways, one 8 feet wide and the other 6 feet wide. meet at right angles. Determine the length of the longest ladder that can be carried horizontally from one hallway to the other.

5. Sketch the graph of the function $f(x) = \frac{x}{x^2-1}$ using the approach presented at the class.

6.

(i) Find the average value of $f(x) = \sqrt{x}$ on the interval $x \in [0, 9]$.
(ii) Find the area of the region bounded by $y = 2\cos x$, $y = \sin 2x$ for $x \in [-\pi, \pi]$.

7.

(i) Let Ω be the region bounded by $y = \sec x$, $x = 0$, $x = \frac{1}{4}\pi$, and $y = 0$. Find the volume of the solid generated by Ω about the x-axis.
(ii) Find the centroid of the bounded region determined by the curves $y = x^2$ and $y = x$.

8. Let

$$f(x) = \int_2^x \sqrt{1+t^2}\, dt.$$

(i) Prove that f has an inverse.
(ii) Find $(f^{-1})'(0)$.

9. Let f be a function such that f' is continuous on $[a, b]$. Show that

$$\int_a^b f(t)f'(t)\, dt = \frac{1}{2}\Big[f^2(b) - f^2(a)\Big].$$

10. Prove the following first mean-value theorem for integrals:

If f is continuous on $[a, b]$, then there is at least one number c in (a, b) for which

$$\int_a^b f(x)\, dx = f(c)(b-a)$$

by using the following mean-value theorem of differential calculus:

If g is differentiable on the open interval (a, b) and continuous on the closed interval $[a, b]$, then there is at least one number c in (a, b) for which

$$g'(c) = \frac{g(b) - g(a)}{b-a}.$$