MIDTERM 1 FOR CALCULUS

Date: 1999, October 28, 4:10–5:00PM

Problem #1 is worth 12 point, #2 and #3 are worth 14 points each, the remainings are worth 10 points each.

1. Compute the following limits:

(i) $\lim_{h\to 0} \frac{1-1/h^2}{1+1/h^2}$

(ii) $\lim_{x\to 0} \frac{1-\cos 4x}{9x^2}$

2. Find the derivative of the following functions:

(i) $f(x) = (x^2 - 1/x^2)^{10}$

(ii) $y = \cos^4 \sqrt{x}$

3.

(i) Find $f''(0)$ where $f(x) = \frac{1}{1-x}$.

(ii) Find $\frac{d}{dx}\big[f\big(\frac{x-1}{x+1}\big)\big]$ in terms of f'.

4. Use implicit differentiation to find dy/dx for $\sin(x+y) = xy$.

5. Find the equations for the tangent and normal lines of the curve $x = \cos y$ at the point $(\frac{1}{2}, \frac{\pi}{3})$.

6. Let

$$f = \begin{cases} A^2x^2, & \text{if } x \le 2; \\ (1-A)x, & \text{if } x > 2. \end{cases}$$

For which values of A is f continuous at 2?

7. An object that weights 150 pounds on the surface of the earth will weight $150(1 + \frac{r}{4000})^{-2}$ pounds when it is r miles above the surface. If the object's altitude is increasing at the rate of 10 miles per second, how fast is its weight decreasing at the instant it is 400 miles above the surface?

8. Use intermediate value theorem to prove that the equation $x^3 + ax^2 + bx + c = 0$ has at least one real root where a, b, c are constant.

9. Let f be twice differentiable on the open interval (a, b) (i.e., $f''(x)$ exists for any $x \in (a, b)$). Prove that if $f''(x) \neq 0$ for all $x \in (a, b)$, then f has at most two zeros in (a, b).