Linear Algebra II, FINAL, Yung-fu Fang, 2011/01/10

Show All Work

1. (a) State some sufficient and necessary conditions for A or T being diagonalizable. 3% each

(b) State the definition of Pseudoinverse. [5%]

(c) State the Singular Value Decomposition. [5%]

(d) State the definition of a vector space. [5%]

(e) State the definition of an inner product. [5%]

(f) State the Cayley-Hamilton Theorem and the definition of minimal polynomial. [5%]

2. Let $\{u_1, u_2, u_3\}$ be a set of linearly independent vectors in an inner product space. Use the Gram-Schmidt Process to compute the orthogonal vectors $\{v_1, v_2, v_3\}$. Then normalize these vectors. [10%]

3. Let $T : R^2 \to R^2$ be the rotation by θ. Prove that T is a linear operator. Is T diagonalizable? Explain! [10%]

4. Let T be a normal operator on a finite-dimensional complex inner product space V. Use the spectral decomposition $T = \lambda_1 T_1 + \lambda_2 T_2 + \cdots + \lambda_k T_k$ to prove that there exists a normal operator U on V such that $U^2 = T$. [10%]

5. Let V and W be finite-dimensional inner product spaces. Let $T : V \to W$ and $U : W \to V$ be linear transformations such that $TUT = T$, $UTU = U$, and both UT and TU are self-adjoint. Prove that $U = T^\dagger$ [10%]

6. Find the singular value decomposition and $A^\dagger$ for $A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \end{pmatrix}$. [10%]

7. Let $A \in M_{3\times 3}$ and diagonalizable with distinct eigenvalues $\lambda_1, \lambda_2, \lambda_3$ and eigenvectors v_1, v_2, v_3 Find $A = QDQ^{-1}$ and $A = U\Sigma V^*$. Express the matrices Q, D, U, Σ, and V in terms of $\lambda_1, \lambda_2, \lambda_3$ and v_1, v_2, v_3. [10%]

8. Let A be a 2×2 matrix. Let $\lambda \in R$ and $\xi \in R^2$ such that $(A - \lambda I)\xi = 0$. Suppose that the null space of $A - \lambda I$ is 1-dimensional and $(A - \lambda I)^2$ is a zero matrix. Show there is a vector $\eta \in R^2$ such that $(A - \lambda I)\eta = \xi$. Show that $\beta = \{\xi, \eta\}$ is an ordered basis for R^2. Find $[L_A]_\beta$. Find the matrices Q and J such that $A = QJQ^{-1}$. [10%]

9. Let $A = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ -1 & -1 & 4 \end{pmatrix}$. Find a Jordan Canonical Form for A and the minimal polynomial of A [10%]

Have A Nice Winter Break!